Analysis of a Physical Internet enabled parking slot management system

Bing Qing Tan[1], Suxiu Xu[2] and Kai Kang[3]

1. School of management, Jinan University, Guangzhou, China
2. Institute of Physical Internet, School of Intelligent Systems Science and Engineering, Jinan University (Zhuhai Campus), Zhuhai, China
3. Department of Industrial and Manufacturing Systems Engineering, The University of Hong Kong, Hong Kong

Corresponding author: tanbingqing0910@stu2018.jnu.edu.cn

Abstract: *This paper considers a Physical Internet enabled parking slot management system where the drivers are matched with two types of parking slots through the Vickrey auction. In order to study the impact of the implementation of a system on the drivers and parking slots, a two-stage stochastic model is developed. Firstly, according to the number of parking slots and their cost function distributions, this paper calculates the expected price and the expected profit in the auction model. After that, a continuous-time Markov chain model is established to evaluate the performance in a dynamic environment, including random arrivals of the Poisson process and possible abandonment of drivers and parking slots. By combining these two models, this paper utilizes a quantitative method to analyze the performance of the system, and our analysis plays an important role in every part of the system in future decisions.*

Keywords: *Physical Internet; Markov chain; Vickrey auction; Performance evaluation*

1 Introduction

Searching for an available parking space in many downtown areas has become a daily concern. The time spent on cruising often constitutes a substantial portion of travel time, because drivers usually keep on cycling the parking area until they found an empty parking space (Liu et al. (2014). Both theoretical and empirical literature substantiates that constantly cruising commonly results in more traffic congestion and air pollution (Anderson and de Palma, 2004; Arnott and Inci, 2006; Glazer and Niskanen, 1992; Glazer et al., 1992; Shoup, 1997, 2006). For example, Shoup (2005) proposes that cruising may considerably inflate overall vehicle travel since it makes trips longer: some parking places in Los Angeles induce 3,000 extra vehicle kilometers per year for cruising, van Ommeren et al. (2012) utilize empirical evidence to investigate cruising for parking, and finds cruising for parking is more common with shopping and leisure than for work-related activities. To alleviate such traffic congestion and improve the convenience for drivers, some studies in parking issues have emphasized the reservation system of parking spaces (Hanif et al., 2010; Hashimoto et al., 2013; Kaspi et al., 2014; Liu et al., 2014).

The difficulty of searching for an available parking space is largely reduced through the smart parking management system. For example, Xiao et al. (2018b) propose a model-based practical framework to predict future occupancy from historical occupancy data alone. Wang and He (2011) develop a new prototype of a reservation-based smart parking system, and implement a parking reservation policy to balance the benefit of service providers and requirements from the users. In fact, most existing studies mainly focus on the architecture and framework of the parking management system. The management problem of parking space is complicated by parking time required for different drivers. Hence, a key aspect of management should be considered is that each parking space can be split into several parking slots. Meanwhile, with the development of the sharing economy, some studies have considered how to realize private parking space sharing (Kong et al., 2018; Xiao et al., 2018a; Xu et al., 2016).

However, little attention has been devoted to the modeling and analysis of the system. There is enormous potential in evaluating and analyzing the performance of the system after modeling. In this paper, we consider combining the auction model with a continuous-time Markov model (CTMM) that capture important characteristics of the system. Also, we further evaluate the performance of the system for possible values of the whole parameters. This research is thus motivated by answering the following questions:

(1) How to develop a simplified analytical model that captures important characteristics of the system?

(2) The available time of parking spaces may not exactly match the needs of drivers. How to allocate and make price when space and drivers choose to abandon the platform?

(3) How to consider performance evaluation from different aspects.

On this basis, we first develop a Physical Internet-enabled parking management system framework that integrates agent and IoT technologies. The system framework facilitates parking space trade between parking space owners and drivers, and also collects diverse data automatically for the effect analysis. Then, we consider there are two types of parking slots: public parking slots and private parking slots that have lower costs for the platform. Drivers prefer the private parking spaces. We also consider there are two types of drivers: who only need one parking slot and who need multiple consecutive parking slots. In order to model and analyze, we develop a general two-stochastic model of a parking slot management platform (PSMP). Firstly, according to the number of parking slots and their cost distributions, this paper calculates the expected price in the auction model. After that, a continuous-time Markov chain model is established to evaluate the performance in a dynamic environment, including random arrivals of the Poisson process and possible abandonment of drivers and parking slots. By combining these two models, this paper utilizes a quantitative method to analyze the performance of the system, and our analysis plays an important role in every part of the system in future decisions.

2 Related study

Parking plays a significant role in urban transport systems, many articles address one or both of modeling and evaluation for the parking problem. Most studies in the modeling category seek to shorten cruising time by giving guidance to motorists in parking systems. Auctions ask and answer the most fundamental questions in economics: who should get the goods and at what prices (Cramton et al., 2007). Hence, there has been a growing interest in using auctions for resource allocation and pricing problems. For instance, Edelman et al. (2007) investigated a new auction mechanism used by search engines to sell online advertising, and they proved this mechanism is an ex-post equilibrium, with the same payoff to all players as dominant strategy equilibrium of VCG mechanism. Two types of auction mechanisms were designed and compared for multi-unit transportation procurement, and allocated carriers to shippers efficiently in logistics e-marketplaces (Huang and Xu, 2013). A reverse iterative combinatorial auction was designed as the allocation mechanism to assign the spectrum resources for device-to-device communications with multiple user pairs (Xu et al., 2013). And in a literature review paper, Lafkihi et al. (2019) summarized a large number of studies that are related to transportation service procurement and further explained why auction is a suitable mechanism for resource allocation and pricing problems. As a kind of necessary resource in our daily life, parking spaces are frequently utilized resources. Recently, more and more researchers applied auctions to efficiently allocate and reasonably make the price of parking spaces. Xiao et al. (2018a) addressed two truthful auction mechanisms to design the parking slot allocation and transaction payment rule based on full consideration of parking time assignment, and they compared two mechanisms to solve the four fundamental shared parking problems. They further propose a fair recurrent double VCG (FRD-VCG) auction mechanism to approach the emerging shared parking management problem, this mechanism has the potential to persuade participants to remain in the market whilst it improves the market's retention rate, the parking slot's utilization rate and the participants' utilities (Xiao and Xu, 2018). Kong et al. (2018) combined O-VCG auction with market design mechanisms, and they integrated this combination into parking space sharing and allocation problems. Tan et al. (2019) designed two sequential auction mechanisms based on first- and second-price auction and they utilized forecasted price to combine these two sequential auction mechanisms with two sharing rules to solve parking space allocation and pricing problem. And an auction-based implementation was proposed, to guarantee congestion-free traffic, and to reduce both the travel time and travel time unreliability. This auction-based highway reservation shows great potential as a new traffic management system (Su and Park, 2015). Shao et al. (2020) considered an auction-based parking reservation problem where a parking management platform can deal with the demand disturbances based on the proposed effective multi-stage Vickrey-Clarke-Groves (MS-VCG) auction mechanism.

3 Physical Internet-enabled urban parking management platform

Figure 1 describes the system architecture of the PI-enabled parking management system. As a classical parking management solution, this system consists of three levels: Infrastructure as a Service (IaaS) level, Platform as a Service (PaaS) level and Software as a Service (SaaS) level.

IaaS level contains hardware and software layers. The hardware layer includes smart sensors, smart gateway, servers and storage, and networks. PI-enabled smart parking environment is established using smart sensors, such as RFID for parking access control, closed-circuit television for parking surveillance and security, and ultrasonic sensors for parking vacancy detection. Physical parking objects, such as parking spaces and security barriers, are converted into smart parking objects. Smart gateway connects, manages and controls smart parking objects. Networks transmit parking-related data in a flexible-to-configure manner, and generally consist of 5G, WiFi, Bluetooth, Transmission Control Protocol/Internet Protocol, Ultra-wideband and Zigbee. In addition, the software layer includes a Gateway Operating Sytem (GOS) and management tools. GOS as a light-weighted middleware system is deployed on desktops, servers and mobile devices to manage smart objects. Smart parking objects are thus managed in different parking scenarios, such as curbside parking areas, public and automated parking garages, and private parking spaces.

PaaS level contains five key components: agent parking space repository, platform service management, parking space agent management, data analytics services, and database. These components facilitate easy deployment and reduce the complexity of managing the underlying hardware and software. Relying on smart parking objects, parking space agents are virtualized and stored in the agent parking space repository. The real-time data are also stored in the database. Moreover, the platform service management module allows the system owners to maintain and configure the system and its services for fulfilling the requirements of stakeholders. The agents are managed by the parking space agent management module for daily parking operations. Data analytics services provide system owners with powerful tools to measure the system performances. Based on the analysis results, the system owners could then propose some strategies and policies to improve the performance and increase the profit.

At the top of this architecture, SaaS level contains various applications for parking management, including parking supervision application, parking space allocation application, parking space pricing application, and parking space navigation application. Various stakeholders, such as drivers and parking space owners, access the applications through SaaS level. Other systems could also gain access to the system. For example, traffic control systems extract parking data to control traffic signals. Also, online payment applications are connected with the system to support online transactions. The applications not only improve driver satisfaction but also allocate parking spaces to drivers with reasonable prices, so that parking space owners make a healthy profit in the market.

In this paper, we aim at the performance analysis of the parking space market by data analytics services considering parking space allocation and pricing services. Furthermore, the PI-enabled parking management system collects required data for analysis, including the arrivals of both parking space owners and drivers, their abandonment and payment records.

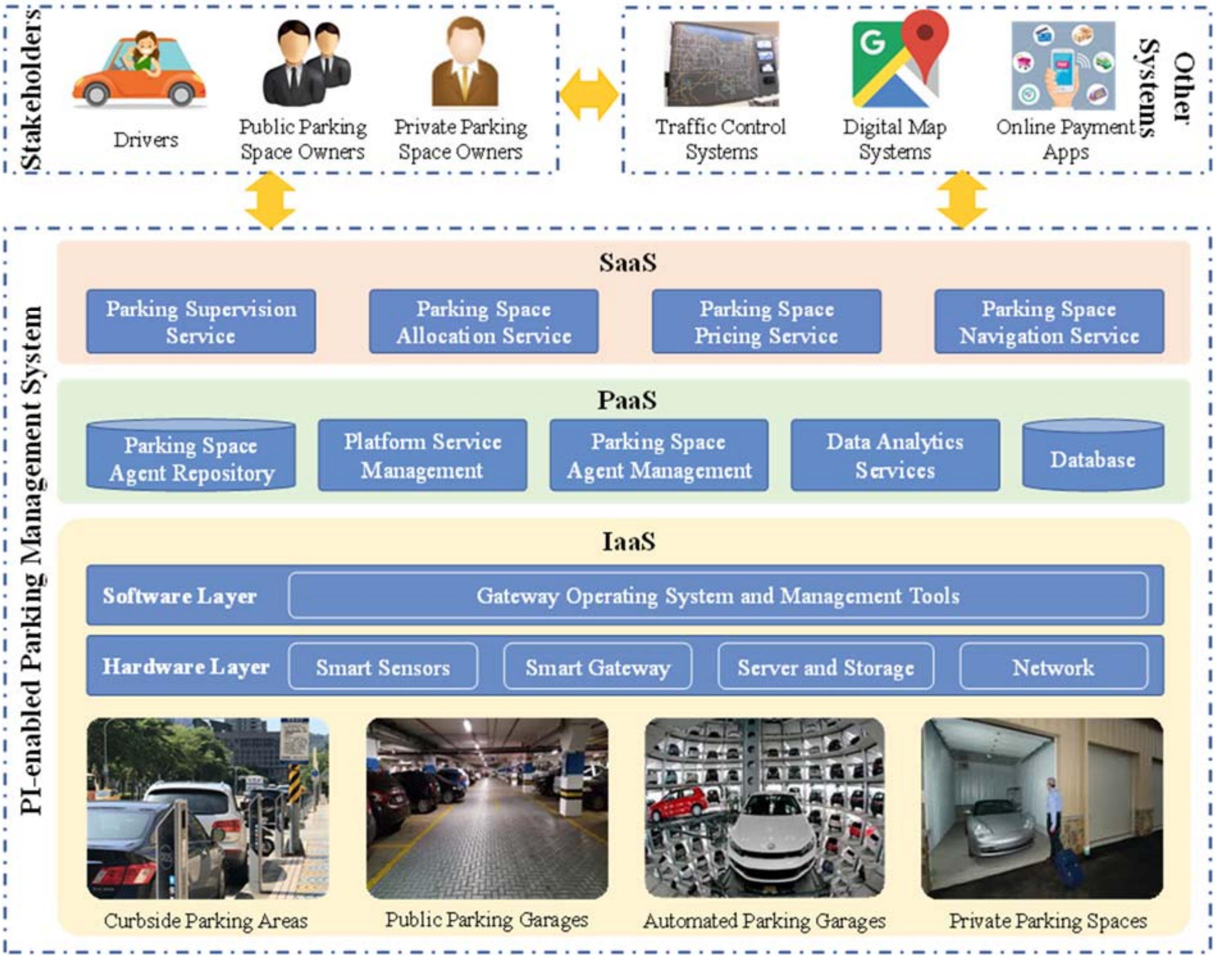

Figure 1: Architecture of the Physical Internet-enabled parking management systems

4 Model

4.1 Problem description

We consider a Physical Internet-enabled parking slot management platform (PSMP) where a number of available parking slots are allocated to drivers. In the platform, there are two types of parking slots that respond to two types of drivers by utilizing auctions to make short-term contracts. Two types of parking slots are public parking slots and private parking slots that have lower costs for the platform. Each parking space is split into several parking slots based on their available time, e.g. 8:00 am to 8:30 am, 4:00 pm to 4:30 pm. Two types of drivers are those who only need one parking slot and who need multiple consecutive parking slots. The parking orders from drivers are matched with parking slots through an electronic auction based on the auction centers (ACs) that are commonly shared among drivers and parking slots. The parking slots with the same available time and in the nearby area are regarded as homogeneous, so each auction center is designed to handle the matching of drivers and parking slots in a specific area. The parking demand is given to the parking space owner who submits the lowest bid. If no parking space is available or no appropriate bid is submitted, the driver will simply withdraw the parking demand from the platform. Also, parking spaces which stay at the platform may be useless after some time if they fail to be allocated to drivers.

4.2 Assumption

The following assumptions are made about the drivers and parking slots.

(1) One type of drivers only need one parking slot i.e. $i = 1$, and another type of drivers require two consecutive parking slots i.e. $i = 2$.

(2) Two types of drivers arrive randomly to the platform following a Poisson process with rate parameter λ_1, λ_2, respectively.

(3) If there are no parking slots are available, two types of drivers randomly abandon following a Poisson process with rate parameter α_1, α_2, respectively.

(4) There is no collusion between drivers.

(5) Two types of parking spaces are public parking spaces and private parking spaces, and the cost of private parking spaces is lower than the cost of public parking spaces, i.e. $c_2 < c_1$ (Xu et al., 2016).

(6) A parking space can be split into several parking slots.

(7) Two types of parking slots arrive randomly to the platform following a Poisson process with rate parameter s_1, s_2, respectively.

(8) If there are no drivers are available, two types of parking slots are randomly failed to allocate following a Poisson process with rate parameter β_1, β_2, respectively.

(9) The per-unit cost c_j of two types of parking slots is drawn independently from a continuously differentiable distribution function $F_j(x)$ from a support $[c_j^-, c_j^+]$ with mean $\tilde{c}_j$, which is common to all the participants. In addition, we assume $c_2^+ < c_1^-$ based on the assumption (5).

4.3 Analysis of auction

Vickrey (Vickrey, 1961) discussed the bidding for a prize in which the highest bidder wins the prize but has to pay the second-highest price. Actually, this single-item second-price sealed-bid auction is equivalent to Vickrey auction. In this paper, the driver with the lowest bid wins the parking slot and is paid the second-lowest price.

Denote $p\left(n_1, n_2, n_{s_1}, n_{s_2}\right)$ as the expected price when n_i drivers and n_{s_j} parking slots are at the platform in steady-state. Let $q\left(n_1, n_2, n_{s_1}, n_{s_2}\right)$ be the expected profit of the winner. In this paper, we consider the cost distribution of private and public parking slots are uniform. That is,

$$F_i(x) = \frac{x - c_i^-}{c_i^+ - c_i^-},\ c_i^- \le x \le c_i^+,\ i = 1, 2.$$

$p\left(n_1, n_2, n_{s_1}, n_{s_2}\right)$ and $q\left(n_1, n_2, n_{s_1}, n_{s_2}\right)$ can be written as

$$p\left(n_1, n_2, n_{s_1}, n_{s_2}\right) = \begin{cases} c_2^- + \dfrac{2(c_2^+ - c_2^-)}{n_{s_2} + 1} & n_1 = n_2 = 0, n_{s_2} > 1 \\ c_1^- + \dfrac{c_1^+ - c_1^-}{n_{s_1} + 1} & n_1 = n_2 = 0, n_{s_1} > 1, n_{s_2} = 1 \\ c_1^- + \dfrac{2(c_1^+ - c_1^-)}{n_{s_1} + 1} & n_1 = n_2 = 0, n_{s_1} > 1, n_{s_2} = 0 \\ & n_1 = n_2 = 0, n_{s_1} = 0, n_{s_2} = 1 \\ c_1^+ & n_1 = n_2 = 0, n_{s_1} = 1, n_{s_2} = 0 \\ & n_1 + n_2 \ge 1, n_{s_1} = n_{s_2} = 0 \end{cases} \tag{1}$$

$$q\left(n_1, n_2, n_{s_1}, n_{s_2}\right) = \begin{cases} \dfrac{c_2^+ - c_2^-}{n_{s_2} + 1} & n_1 = n_2 = 0, n_{s_2} > 1 \\ c_1^- + \dfrac{c_1^+ - c_1^-}{n_{s_1} + 1} - \tilde{c}_2 & n_1 = n_2 = 0, n_{s_1} > 1, n_{s_2} = 1 \\ \dfrac{c_1^+ - c_1^-}{n_{s_1} + 1} & n_1 = n_2 = 0, n_{s_1} > 1, n_{s_2} = 0 \\ c_1^+ - \tilde{c}_2 & n_1 = n_2 = 0, n_{s_1} = 0, n_{s_2} = 1 \\ c_1^+ - \tilde{c}_1 & n_1 = n_2 = 0, n_{s_1} = 1, n_{s_2} = 0 \end{cases} \tag{2}$$

4.4 Analysis of the arrival-departure processes

The previous analysis has shown that the auction price and the profit depend on the number of drivers and the number of parking slots. Since the number of drivers and parking slots change dynamically with the random arrival-departure process of drivers and parking slots, we then examine the dynamics of the system to determine the steady-state distribution of the number of drivers and the number of parking slots at the platform.

Denote the state of the auction center at time t by $S(t) = (N_1(t), N_2(t), N_{s_1}(t), N_{s_2}(t))$. The process $\{S(t), t \ge 0\}$ is a continuous-time Markov Chain, because the inter-arrival and departure times of drivers and parking slots are exponentially distributed. This birth-death process is ergodic and has a stationary distribution. The steady-state probabilities are defined as

$$\pi(n_1,n_2,n_{s_1},n_{s_2}) = \lim_{t\to\infty}\Pr\{N_1(t)=n_1, N_2(t)=n_2, N_{s_1}(t)=n_{s_1}, N_{s_2}(t)=n_{s_2}\} \quad (3)$$

In order to solve the model, the state-transition equations are obtained and the steady-state probabilities are calculated by truncating the state slot at state $\left(K_1, K_2, K_{s_1}, K_{s_2}\right)$. K_1 is the maximum number of type 1 drivers that can be accepted by the platform, K_2 is the maximum number of type 2 drivers that can be accepted by the platform, K_{s_1} is the maximum number of type 1 parking slots arrive at the platform, and K_{s_2} is the maximum number of type 2 parking slots arrive at the platform. By setting sufficiently large values of K_1, K_2, K_{s_1} and K_{s_2}, the rejection probability for drivers and parking slots can be negligible. In addition, there are a total of $(K_1+1)(K_2+1)+(K_{s_1}+1)(K_{s_2}+1)-1$ states in the resulting state $S(t)=(N_1(t), N_2(t), N_{s_1}(t), N_{s_2}(t))$.

5 Numerical study

5.1 Parameter setting

We assume $c_2^- = \tau c_1^-$, $c_2^+ = \tau c_1^+$ and $c_1^- = \varepsilon c_1^+$. Therefore, the distributions are defined in the range $[\varepsilon c_1^+, c_1^+]$ and $[\tau\varepsilon c_1^+, \tau c_1^+]$, the mean cost $\tilde{c}_1 = \frac{(1+\varepsilon)c_1^-}{2}$ and $\tilde{c}_2 = \frac{(1+\varepsilon)\tau c_1^+}{2}$. In order to ensure $c_2^+ < c_1^-$ holds, τ should be less than ε. And we set capacity for two types of drivers and two types of parking slots $K_1 = K_2 = 5$ and $K_{s_1} = K_{s_2} = 6$. We evaluate the performance of the system as indexes vary on the following two indexes for different λ_1, α_2, s_1, β_2, τ and ε: (1) $\bar{P}$: the mean of expected price $\bar{P}$; (2) $\bar{Q}$: the mean of expected profit.

5.2 Results

Drivers and parking spaces are traded at market price c_1^+ without PSMP. The auction center of PSMP is expected to lower the price paid by drivers. Table 1 shows the mean of the expected price paid by drivers for different type 1 driver and type 1 parking slots arrival rate, type 2 driver and type 2 parking slot abandonment rate, and the different cost distributions. Table 1 expresses that as the type 1 parking slots arrival rate s_1 increases, the number of parking slots participating in the auction will increase, which will lead to a decrease in the expected price. After the number of parking slots no longer increases, as the type 1 driver arrival rate λ_1 increases, the expected price will increase. Meanwhile, if the abandonment rate α_2 of type 2 driver increases, the total number of drivers remaining to continue bidding for parking spaces will decrease, which will result in a lower expected price. The larger the type 2 parking slot abandonment rate α_2, the smaller the number of available parking slots in the system, and the result is a significant increase in the price. Especially when $\alpha_2 = 2$, the expected price exceeds 0.9 which is close to the market price $c_1^+ = 1$.

Table 1: The mean of the expected price

$\bar{P}$	λ_1				
s_1	0.2	0.3	0.4	0.5	0.6
1.2	0.21416	0.33214	0.45765	0.58254	0.69941
1.4	0.1876	0.29421	0.41	0.52838	0.64273
1.6	0.1675	0.26423	0.37062	0.48165	0.59168
1.8	0.15219	0.24061	0.33835	0.44183	0.54645
2	0.14038	0.22197	0.31207	0.40825	0.50693
$\bar{P}$	α_2				
β_2	0.1	0.2	0.3	0.4	0.5
0.4	0.48034	0.47221	0.46551	0.45997	0.45534
0.6	0.61336	0.60482	0.59767	0.59168	0.58663
0.8	0.72658	0.7189	0.71238	0.70686	0.70215
1	0.82367	0.81777	0.8127	0.80835	0.80459
1.2	0.9078	0.90431	0.90127	0.89862	0.89628
$\bar{P}$	ε				
τ	0.7	0.75	0.8	0.85	0.9
0.2	0.515	0.51097	0.50693	0.50328	0.4995
0.3	0.51742	0.51294	0.50847	0.50441	0.50022
0.4	0.51984	0.51492	0.51	0.50554	0.50093
0.5	0.52225	0.51689	0.51153	0.50668	0.50165
0.6	0.52467	0.51887	0.51307	0.50781	0.50237

Table 2 analyzes the effect of different type 1 driver and type 1 parking slots arrival rate, type 2 driver and type 2 parking slot abandonment rate, and the different cost distributions on the mean of the expected profit. Since the reverse auction is utilized, when type 1 driver arrival rate λ_1 increases, the demand increases. Therefore, when the number of parking slots participating in the auction remains unchanged, the larger λ_1, the greater the mean of the expected profit $\bar{Q}$. Also, as the type 1 parking slots arrival rate s_1 increases and λ_1 remains unchanged, which will lead to a decrease in the expected profit. When the abandonment rate α_2 increases, there are still type 1 parking slots in the system that can participate in the auction, since the change in α_2 has no obvious impact on the expected profit. In addition, when we set $\beta_2 = 2\beta_1$, as the two types parking slots' abandonment rates β_1 and β_2 both increase, the number of available parking slots in the system will decrease, which will lead to a decline in the expected profit.

Table 2: The mean of the expected profit

$\bar{Q}$	λ_1				
s_1	0.2	0.3	0.4	0.5	0.6
1.2	0.52482	0.53683	0.54965	0.56172	0.57235
1.4	0.49333	0.50807	0.52073	0.532	0.54188
1.6	0.46428	0.48249	0.49566	0.50642	0.5156
1.8	0.43782	0.4595	0.47357	0.48415	0.49274
2	0.41415	0.4388	0.45391	0.46453	0.47272
$\bar{Q}$	α_2				
β_2	0.1	0.2	0.3	0.4	0.5
0.4	0.52314	0.52395	0.52495	0.52609	0.52735
0.6	0.51445	0.51462	0.51502	0.5156	0.51633
0.8	0.50909	0.50878	0.50872	0.50888	0.50921
1	0.50568	0.50499	0.50457	0.50439	0.5044
1.2	0.50344	0.50245	0.50174	0.50128	0.50103
$\bar{Q}$	ε				
τ	0.7	0.75	0.8	0.85	0.9
0.2	0.5316	0.50216	0.47272	0.44604	0.41844
0.3	0.48527	0.45583	0.4264	0.39972	0.37212
0.4	0.43895	0.40951	0.38007	0.35339	0.32579
0.5	0.39262	0.36318	0.33374	0.30706	0.27947
0.6	0.34629	0.31686	0.28742	0.26074	0.23314

6 Conclusion

The contribution of this paper is to consider a Physical Internet-enabled parking slot management system where a number of available parking slots are allocated to drivers in the truthful reverse auction price. And the system performances are analyzed by a continuous-time

Markov chain. Our results show how the arrival rate of drivers and parking slots, and the abandonment rate of drivers and parking slots affect the expected price and the expected profit.

In addition, even though we focus on the parking slot management system, our model and results can be utilized in other situations such as online auction and transportation service procurement auctions where the number of bidders varies randomly. Moreover, combining auction and continuous-time Markov Chain can be applied to evaluate other resource management systems.

References

- Anderson, S.P., de Palma, A., (2004): The economics of pricing parking. Journal of Urban Economics, v55, no1, 1-20.
- Arnott, R., Inci, E., (2006): An integrated model of downtown parking and traffic congestion. Journal of Urban Economics, v60, no 3, 418-442.
- Cramton, P., Shoham, Y., Steinberg, R., (2007): An overview of combinatorial auctions. ACM SIGecom Exchanges, v7, no1, 3-14.
- Edelman, B., Ostrovsky, M., Schwarz, M., (2007): Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. American economic review, v97, no1, 242-259.
- Glazer, A., Niskanen, E., (1992): Parking fees and congestion. Regional science and urban economics, v22, no1,123-132.
- Glazer, A., Niskanen, E., Economics, U., (1992): Parking fees and congestion. Regional Science and Urban Economics, v22, no1, 123-132.
- Hanif, N.H.H.M., Badiozaman, M.H., Daud, H., (2010): Smart parking reservation system using short message services (SMS), 2010 International Conference on Intelligent and Advanced System.
- Hashimoto, S., Kanamori, R., Ito, T., (2013): Auction-Based Parking Reservation System with Electricity Trading, 2013 IEEE 15th Conference on Business Informatics, pp. 33-40.
- Huang, G.Q., Xu, S.X., (2013): Truthful multi-unit transportation procurement auctions for logistics e-marketplaces. Transportation Research Part B: Methodological, v47, 127-148.
- Kaspi, M., Raviv, T., Tzur, M., (2014): Parking reservation policies in one-way vehicle sharing systems. Transportation Research Part B: Methodological, v62, 35-50.
- Kong, X.T.R., Xu, S.X., Cheng, M., Huang, G.Q., (2018): IoT-Enabled Parking Space Sharing and Allocation Mechanisms. IEEE Transactions on Automation Science and Engineering, v15, no 4, 1654-1664.
- Lafkihi, M., Pan, S., Ballot, E., (2019): Freight transportation service procurement: A literature review and future research opportunities in omnichannel E-commerce. Transportation Research Part E-Logistics and Transportation Review, v125, 348-365.
- Liu, W., Yang, H., Yin, Y., (2014): Expirable parking reservations for managing morning commute with parking space constraints. Transportation Research Part C: Emerging Technologies, v44, 185-201.
- Shao, S., Xu, S. X., Yang, H., Huang, G. Q., (2020): Parking reservation disturbances. Transportation Research Part B: Methodological, v135, 83-97.
- Shoup, D.C., 1997. The High Cost of Free Parking. Journal of Planning Education and Research, v17, 3-20.
- Shoup, D.C., (2006): Cruising for parking. Transport Policy 13, no6, 479-486.

- Su, P., Park, B.B., (2015): Auction-based highway reservation system an agent-based simulation study. Transportation Research Part C: Emerging Technologies, v60, 211-226.
- Tan, B.Q., Xu, S.X., Zhong, R., Cheng, M., Kang, K., (2019): Sequential auction based parking space sharing and pricing mechanism in the era of sharing economy. Industrial Management & Data Systems, v119, no8, 1734-1747.
- van Ommeren, J.N., Wentink, D., Rietveld, P., (2012):. Empirical evidence on cruising for parking. Transportation Research Part A: Policy and Practice, v46, no1, 123-130.
- Wang, H., He, W., (2011): A Reservation-based Smart Parking System, 2011 IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS). IEEE.
- Xiao, H., Xu, M., (2018): How to restrain participants opt out in shared parking market? A fair recurrent double auction approach. Transportation Research Part C: Emerging Technologies, v93, 36-61.
- Xiao, H., Xu, M., Gao, Z., (2018a): Shared parking problem: A novel truthful double auction mechanism approach. Transportation Research Part B: Methodological, v109, 40-69.
- Xiao, J., Lou, Y., Frisby, J., (2018b): How likely am I to find parking? – A practical model-based framework for predicting parking availability. Transportation Research Part B: Methodological, v112, 19-39.
- Xu, S.X., Cheng, M., Kong, X.T.R., Yang, H., Huang, G.Q., (2016): Private parking slot sharing. Transportation Research Part B: Methodological, v93, 596-617.
- Xu, C., Song, L., Han, Z., Zhao, Q., Wang, X., Cheng, X., Jiao, B., (2013): Efficiency Resource Allocation for Device-to-Device Underlay Communication Systems: A Reverse Iterative Combinatorial Auction Based Approach. IEEE Journal on Selected Areas in Communications, v31, no9, 348-358.
- Xu, S.X., Cheng, M., Kong, X.T.R., Yang, H., Huang, G.Q., (2016): Private parking slot sharing. Transportation Research Part B: Methodological, v93, 596-617.