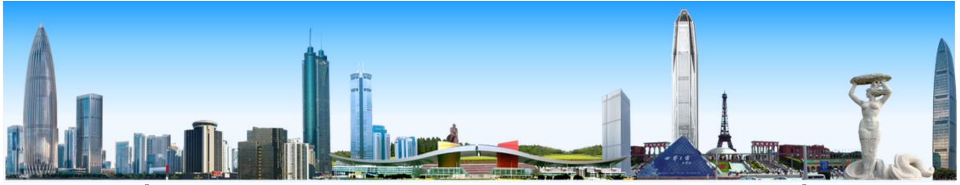

Hierarchical Staffing Problem in Nursing Homes

Ting Zhang[1], Shuqing Liu[2]*, Ping Feng[1], Yali Zheng[1], Wenge Chen[2]
1 Guangdong Rail Transit Intelligent Operation and Maintenance Technology Development Center,Shenzhen Technology University, Shenzhen 518118, China
2 Guangdong University of Technology, Guangzhou 510006, China
Corresponding author: liushuqing@mail2.gdut.edu.cn

Abstract: *With the increasing trend of the aging population, nursing homes have gradually become more and more important in society. Nursing work has the characteristics of "multiple shifts, high time-varying demand, hierarchical and collaborative". Multiple shifts indicates multiple shifts can cover the same time period in 24h of a day; high time-varying demand means the demand in each period may change greatly in a day; hierarchical indicates nurses have different levels; collaborative means employees with different levels cooperate to serve the elderly. This paper involves the shift design problem(SDP) and hierarchical staffing problem, and a two-stage modeling method is adopted. We will design shifts and determine the number of nurses needed for each shift in the first stage and the number of nurses with different levels for each shift is further determined in the second stage. Besides, we also made some sensitivity analysis about the impacts of different matching ratios on the total costs, which obtained some useful conclusions. The results can provide effective enlightenment and rich significance to solve practical problems.*

Keywords: *Aging population; Nursing homes; Shift design; Hierarchical staffing; Healthcare; Collaborative; Sensitivity analysis.*

1 Introduction

With the increase of the aging population in society, the number of the elderly people is also increasing fast in our country. The primary choice for the elderly is home-based care and nursing homes, but it is obvious that the home-based care can't meet the current increasing demand, and the nursing institutions have gradually developed into the rigid needs of the society. However, the workload of nursers who are specialized for the elderly is very heavy and their wages are low. Therefore, many young people, especially those with education, are not willing to work in nursing homes. As a result, it is difficult to recruit employees in nursing institutions, and there is often a shortage of staff when allocating employees. On the other hand, it's also very hard for the elderly employees to bear such a heavy workload. Therefore, the service quality will be declined due to the poor service and staff shortage, which will further reduce the number of customers and the incomes of the nursing homes. This will also lead to the reduction of wages paid to employees, and the nursing homes will fall into the dilemma of recruitment difficulties, and then form a vicious circle.

According to our investigation, the professional nurses who take care of the elderly in nursing homes also have the following characteristics: (1)the number of nurses required within 24 hours a day will fluctuate greatly according to different time periods; (2)multiple shifts can cover the same time period in a day; (3)employees will be classified according to their responsibilities and abilities, and employees with higher levels cost more; (4)due to the different levels of nurses can provide different service, we need different levels of nurses to cooperate to serve the elderly people, and there is a matching ratio between senior employees and junior employees, which means the number of junior employees that a senior employee can lead can't exceed the matching ratio. According to the above characteristics, we can find that the nursing staff problem in nursing homes is a complex combination optimization problem. However, most of the nursing institutions are still manual scheduling at present, which not only takes more time, but also has low efficiency. Moreover, manual scheduling is easy to cause lacking of employees in peak period and surplus of employees in low peak period, which usually leads to the waste of human resources and cost. Unreasonable staffing has become a common problem in most nursing institutions, and it will aggravate the problem of recruitment difficulties in nursing homes. Therefore, an efficient and systematic method is urgently needed to solve the staffing problem in nursing homes.

Shift design problem involves in many fields, such as bank, call center, etc. Musliu et al. studied the shift design problem according to the working characteristics of call center and bank respectively. They divided a day into multiple periods according to the time period, and considered the problems of overstaffing and understaffing in each period. There are also some shift design problems including break windows, Aykin et al. designed flexible rest windows to meet employees' rest time and reduce cost. However, these shift designs seldom consider that employees can be divided into different levels, and the situation that senior employees can lead junior employees was not considered, which must be involved in our problems.

Personnel scheduling problem and task scheduling problem have similarities, and they are also very common in real situations. Seckiner et al. divided the employees into different levels, but their problems didn’t consider the high time-varying demand factors, and the rest was scheduled according to the daily needs. Moreover, these problems only considered the situation that the senior employees can substitute for junior employees, but not vice versa. However, our problems should consider the cooperation of employees at different levels to work for the elderly. Noberto et al. studied the task scheduling problem, and these problems classified employees according to their abilities. But a task should be completed by employees with the same ability instead of the cooperation between employees with different abilities, which is also different from our problem.

The shift design problem was first proposed by Dantzig and was solved by set covering method, and then some researchers used the implicit modeling method to solve the problem and made comparisons with the set covering method. The results showed that the implicit modeling method was much better than the set covering method. With the complexity of the problems, more and more scholars used two-stage modeling method to solve problems. Sana et al. showed how to use a two-stage method to solve the shift design problem and task scheduling problem in detail. Two-stage modeling method can simplify the problem and presents the model more clearly. Among these papers, Lequy et al. used heuristic rules in two stages, and Pakpoom suggested to use genetic algorithm to solve personnel scheduling problem. Besides, Sana also proved the advantages of the method in terms of the quality of the solution and the calculation time.

To sum up, the nursing work in nursing homes has the characteristics of high time-varying demand, multiple shifts, hierarchical and collaborative. In this paper, we will solve the staffing problem under these characteristics, and analyze the impact of different matching ratios on the total costs. This paper will be carried out according to the following sections. Section 2 describes the problem of nursing staffing in nursing homes and puts forward some assumptions. Section 3 establishes a two-stage model to solve the problem according to the nursing characteristics, and introduces the two-stage method and the models in detail. According to different demand distribution situations, some cases about different matching ratios will be tested in section 4, and the experimental results are displayed. Some practical management enlightenment is obtained according to the results. Finally, section 5 concludes this article and puts forward some prospects.

2 Problem definition

At present, nursing homes are facing a difficulty of recruitment due to the characteristics of nursing work, which further leads to a poor service quality and the reduction of customers, and then decrease the incomes. At the same time, manual scheduling is aggravating this problem, so the main problem that the nursing institutions need to solve is the unreasonable staffing problem. Here are some assumptions about this problem:

1. The shift must be started and ended within the specified time, and shifts have different types according to start time, such as morning shift, day shift, afternoon shift and night shift.

2. Each shift has a shift length which should be within 4h-8h, and an employee can only work one shift a day.

3. Multiple shifts are needed and can cover the same time period each day.

4. Different demands of 24 hours may be different, but the demand of each time period should be met to avoid understaffing.

5. Employees have different levels, and the service quality that a senior worker can provide is better than that of junior employees.

6. Junior employees can not be arranged if the shift only has one employee, and they must be led by the employees with the highest level. However, the number of junior workers that a senior worker can lead should be limited.

7. Since different employees have different levels and abilities, hierarchical collaboration should be considered to provide services for the elderly.

8. Different employees have different costs. The cost of an employee includes the part of shift length and the part of the level of the employee, the employee whose level is higher and shift length is longer needs more cost.

It can be seen from the above assumptions that the nursing staff problem belongs to a more complex combinatorial optimization problem. Firstly, because the nursing work has the characteristics of high time-varying demand and multiple shifts, the demand will change dramatically according to different time periods within 24 hours a day, which usually causes poor service quality due to staff shortage and waste of cost and human resources on account of overstaffing. Therefore, we need to design reasonable shifts. On the other hand, due to the characteristics of hierarchical and collaborative, the workers will be divided into several levels according to their abilities, and different levels of employees will cooperate with each other to serve the elderly, which means there have many different collocations when staffing employees. Manual scheduling is obviously inefficient. Therefore, we also need to consider the constraints of hierarchical allocation for staffing. Although these problems are very prominent in nursing homes, they still haven’t found an effective way to solve them.

3 Two-stage modeling

3.1 Two-stage method

In this paper, according to the characteristics of nursing workers in nursing homes, a two-stage modeling method is proposed to solve the problem, because the two-stage model can express the problem more clearly. On the one hand, due to the high time-varying demand and multiple shifts characteristics, it is necessary to design shifts to reduce the waste of personnel resources. In this stage, the shifts will be designed by using genetic algorithm, and the detailed time arrangement and the number of employees in each shift will be determined. On the other hand, the characteristics of hierarchical and collaborative require us to consider the cooperation and collocation of employees at all levels to reduce costs as much as possible. We will take the shift arrangement designed in the first stage as an initial condition, and further determine the final specific staffing of employees at all levels according to the constraints of hierarchical collocation by using heuristic rules. Finally, this paper also tests the influence of different matching ratios of senior employees and junior employees on the total costs under different demand distributions, and obtains some management enlightenment for practical decision-making through sensitivity analysis.

3.2 Parameter definition

For the problem in this article, we have the following definitions:

1. t stands for the start time of each shift, t=0,1,..., 23. The shift should be started within the specified time. Besides, we use h to represent the shift length, whose value is between 4 and 8. If t is 0 and h is 4, it means the shift starts at 0 o 'clock in the evening with a length of 4 hours.

2. S represents each shift, and S_{th} means a shift whose start time is t and shift length is h.

3. K stands for the shift type, when k takes 1, 2, 3 and 4, it means morning shift, day shift, afternoon shift and night shift respectively. Besides, d_k means the total number of the shift type corresponding to k.

4. Each time period of a day in 24 hours were represented by i, i =0,1..., 23. For example, when i takes 0, the time period it represents is from 0 p.m. to 1 a.m.

5. We use B to represent the personnel demand, and B_i stands for the demand in each time period, where i =0,1..., 23, and B_{imax} represents the maximum demand in 24 periods of a day.

6. m stands for the level of employees, m=1,2, ... ,m_x. There are m_x levels among employees in total, and the smaller the value of m, the higher the level of the employee. That means when m takes 1, it represents that the employee is in the highest level. In addition, we assume that m_a is the lowest level of the employee.

7. W represents the number of employees, W_{th} means the total number of employees in the shift whose start time is t and shift length is h, and W_{mth} means the total number of workers in the level m and in the shift whose start time is t and shift length is h.

8. C represents the cost, C_h represents the cost of a worker whose shift length is h, and C_{mh} means the cost of a worker whose level is m and the shift length is h.

9. The matching ratio of senior and junior employees is represented by n, and only the employees with the highest level can lead junior employees. Besides, the number of junior workers that a senior worker can lead is limited. For example, when n takes 3, it means a senior worker can lead at most 3 junior workers.

3.3 Models

According to the characteristics of the problem, a two-stage method is adopted and the problem is divided into two parts, which means the models are established in two stages. In the first stage, the shift design is mainly carried out to reduce the cost of human resources as much as possible under the condition of meeting each time demand. In this stage, the cost is only related to the shift length, and the main consideration is the start time and the length of the shift. In the second stage, the shifts designed in the first stage should be taken as an initial condition, and the hierarchical collaboration should be considered on this basis to reduce the total cost as much as possible. The total cost in this stage is not only related to the shift length, but also related to the level of employees. The costs required by high-level employees and low-level employees are different, and the model is established by combining the constraints of different levels of personnel cooperation. The detailed two-stage models are as follows:

(IP1) $\min \sum_{t,h} C_h W_{th} S_{th}$

$$\sum_{t,h} S_{th} W_{th} \geq B_i \quad (i = 0,1, \ldots ,23) \tag{1}$$

$$S_{th} = \begin{cases} 1 & \text{when the shift } S_{th} \text{ includes the time period } i \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{t,h \in k} S_{th} \leq d_k \ (\forall k) \tag{2}$$

$$W_{th} \leq B_{imax} (\forall t, \forall h) \tag{3}$$

$d_k, W_{th}, b_i \geqslant 0$ and all integer

(IP2) $\min \sum_{m,t,h} C_{mh} W_{mth}$

$$\sum_{m} W_{mth} = S_{th} W_{th} (\forall S_{th} = 1) \tag{4}$$

$$W_{mth} \geq 1 (\forall m; \forall W_{th} \geq m_x) \tag{5}$$

$$W_{mth} = 0 (\forall W_{th} = 1, m = m_a) \tag{6}$$

$$W_{mth} \leq nW_{1th} (\forall W_{th} \geq 1, m = m_a) \tag{7}$$

$W_{mth} \geqslant 0$ and all integer

In integer programming model IP1, the objective function is to minimize the total cost of personnel resources, and the level of the employee isn't considered at this stage. While in integer programming model IP2, although the objective function also aims to minimize the total cost of human resources, different levels of employees are considered in this stage, and the total cost of an employee is also different from the first stage. Among these constrains, constrains (1) means that the arrangement of shifts must meet the need of each time period to avoid understaffing in peak period. Constrains (2) means that the number of different shift types has to be limited. Different values of k stand for different shift types, and the number of shift type which is corresponding to k can't exceed d_k. Constraint (3) shows that the total number of employees in each shift has a upper limit, which cannot exceed the maximum value in 24 periods of a day. Constrains (4) means that the total number of employees at different levels should be equal to the total number of employees on the same shift designed in the first stage. Constrains (5) indicates that there must be at least one employee in each level when the shift has enough employees. Constrains (6) indicates that junior employees can't be arranged if the shift only has one employee. Constrains (7) means that the number of junior employees that a senior employee can lead is limited, which can't exceed n.

In the first stage, we use genetic algorithm to design shift and determine the number of employees in each shift by generating a matrix composed of the start time and the length of a shift. Each value in the matrix stands for the number of the employees in corresponding shift, and iteration is carried out to find the optimal solution. In the second stage, we use heuristic rules to determine the number of employees at different levels in each shift, such as giving priority to the lower cost collocation. For the two problems in different two stages, we use different methods to solve and iterate between the two stages to find a better solution.

4 Experiments

4.1 Parameter setting

According to the models, we set some parameters to test:

1. The shift length of one hour is 10, and then four hours cost 40 and so on.

2. The number of each shift type shouldn't exceed 3, including morning shift, day shift, afternoon shift and night shift. Table 1 shows the detailed time arrangement of the shift type.

3. Employees are divided into three levels: senior, intermediate and junior, which means the value of m_x is 3. When m takes 1, it represents the senior employee, which is also means the highest level. When m takes 3, it represents the junior employee, which means m_a is 3. The cost of a junior, intermediate and senior worker is 50, 70 and 100 respectively, and the total cost of a worker (C_{mh}) is equal to the sum of the worker's level cost and the shift length's cost.

4. We assume that the number of junior employees that a senior employee can lead could be 1, 2 or 3, which means there are 3 different situations. A senior employee can be allowed to lead at most 1, 2 or 3 junior employees respectively.

5. The demand of each time period(B_i) is irregular, and we will test the demand curves with a slow and sharp fluctuation in the case of unimodal, bimodal and trimodal respectively. In addition, we also test the case that the demand fluctuation satisfies Poisson distribution and binomial distribution, and analyze the impact of different matching ratios on the total costs under these different demand conditions.

Table 1: Time arrangement of each shift type

Shift type	Min-start	Max-start	Min-length	Max-length
Morning shift	06:00	08:00	04:00	08:00
Day shift	09:00	11:00	04:00	08:00
Afternoon shift	12:00	16:00	04:00	08:00
Night shift	22:00	00:00	04:00	08:00

4.2 Results

Since the demand of each time period must be an integer, and the characteristics of high time-varying demand may lead to different needs in different periods. In addition, the different cases of demand can be divided into unimodal, bimodal and trimodal according to the number of peaks, and the fluctuation of each peak situation can be different. We keep the maximum peak value and the minimum peak value fixed in different peak conditions, which is 30 and 5 respectively. The fluctuation situation can be divided into two types: slow and sharp. We will test the impact of different matching ratios on the total costs under different peaks and fluctuations. At the same time, the demand curve with constant fluctuation is taken as the reference curve, and the results are compared and analyzed. In addition, Poisson distribution and binomial distribution are often used to solve the problem of meeting customer demands in reality, so we also test the influence of different matching ratios on the total costs when the demand satisfies Poisson distribution and binomial distribution.

When the fluctuation is sharp or slow, the results of the total cost affected by the matching ratios under different peak conditions are shown in Figure 1 and Figure 2 respectively.

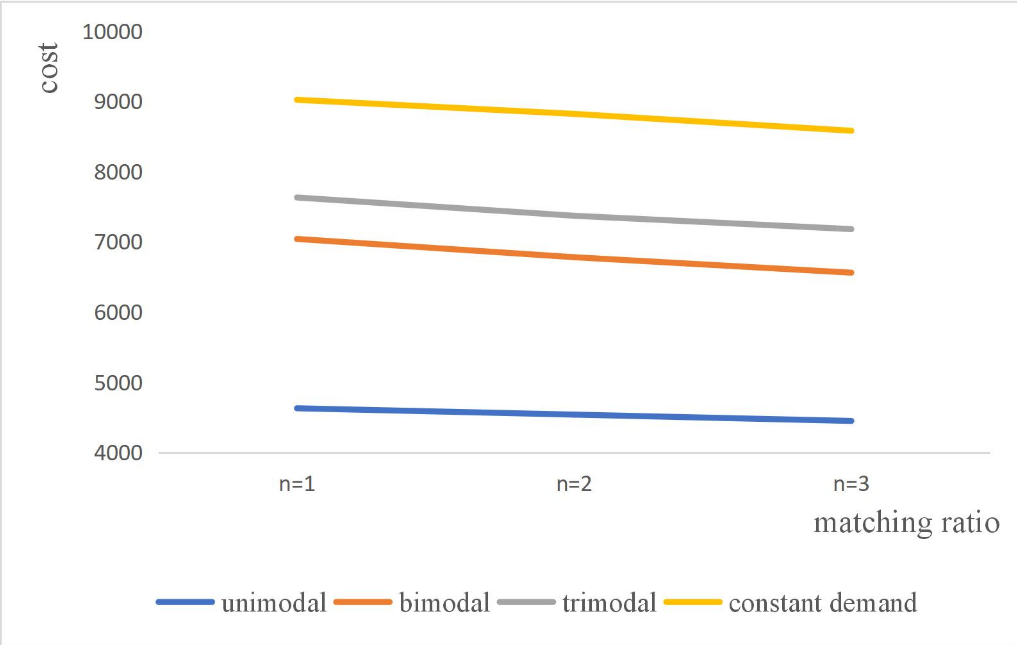

Figure 1: The influence of different matching ratios on the total costs under different peak conditions when the fluctuation is sharp

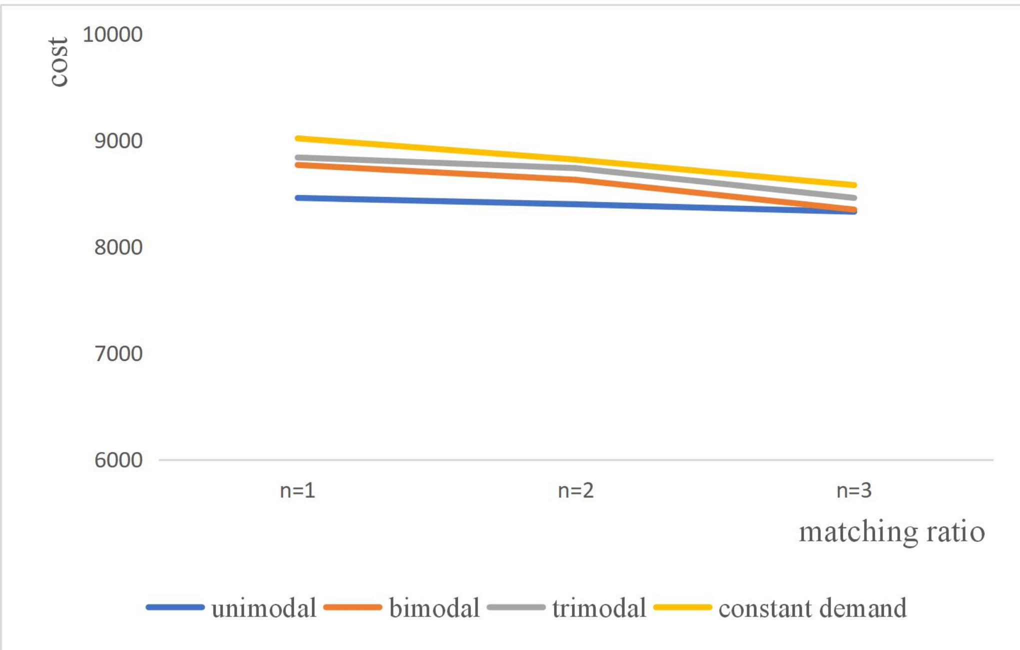

Figure 2: The influence of different matching ratios on the total costs under different peak conditions when the fluctuation is slow

According to the changes of the total costs in the above figures, the following conclusions can be drawn:

1. When the fluctuation differs greatly, the total costs of unimodal demand changes more obviously, and the growth rate of total costs of bimodal is larger than that of trimodal.

2. No matter what the fluctuation is, the total cost of unimodal demand is almost unchanged with the increase of the matching ratio. That means there is tiny differences among the total costs with the increase of matching ratio in the case of unimodal.

3. With the increase of the matching ratio, the total costs of bimodal and trimodal both show a decreasing trend. However, when the fluctuation is slow, the decreasing trend is nonlinear, and the total costs of bimodal and trimodal decrease more obviously when n increases from 2 to 3. While when the fluctuation is sharp, the decreasing trend is almost linear. The total costs of bimodal and trimodal decrease almost linearly when n increases from 1 to 3.

When the demand satisfies the Poisson distribution, we test the impact of different matching ratios on the total costs under different mean values. The results are shown in Figure 3.

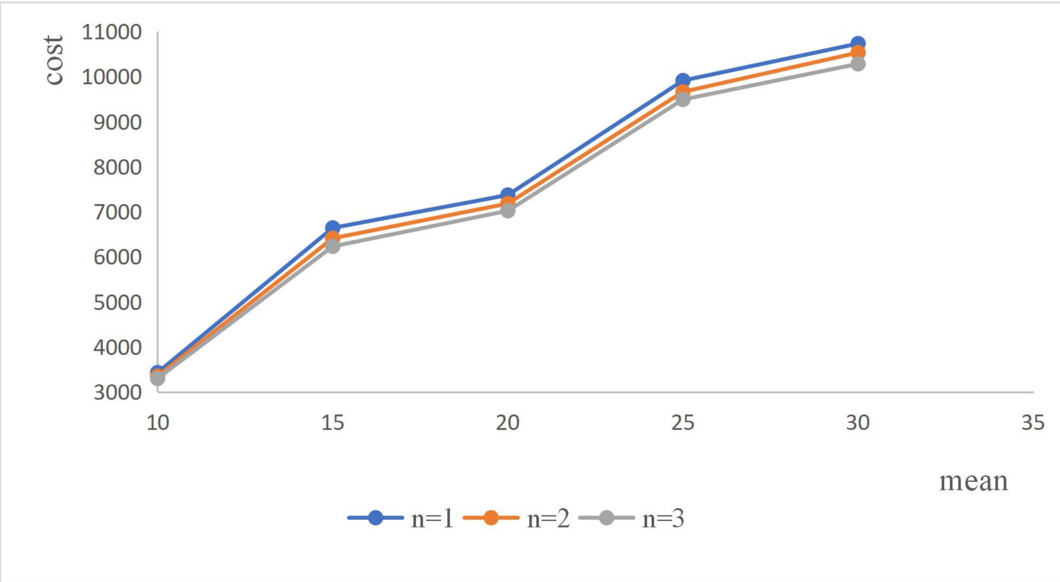

Figure 3: The influence of different matching ratios on the total costs when the demand satisfies the Poisson distribution

From this figure, we can find that in Poisson distribution, with the increase of the mean value, the total costs will gradually increase, but under the same mean value, the total cost does not differ significantly under different matching ratios. Moreover, with the increase of the mean value, the influence of different matching ratios on the total costs doesn't show an obvious change trend. In other words, even if the mean value is changed, the total cost is almost not affected by different matching ratios when the demand satisfies Poisson distribution.

When the demand satisfies the binomial distribution, we test the influence of different matching ratios on the total costs under two situations. One situation is different mean values with a fixed variance of 0.5, and the other situation is different variances with a fixed mean value of 30. The results are shown in Figure. 4 and Figure. 5 respectively.

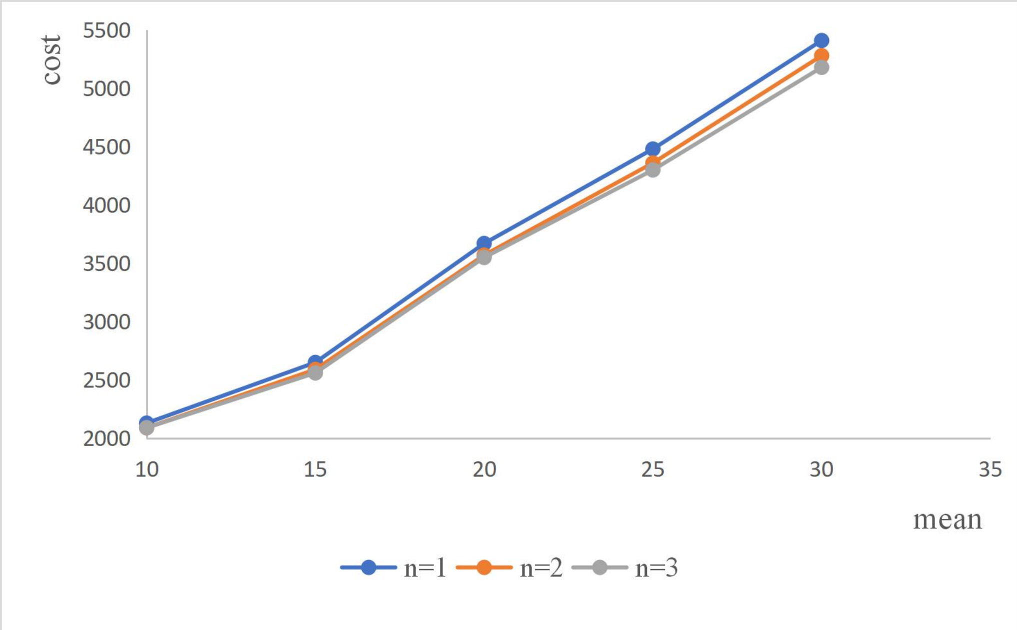

Figure 4: The influence of different matching ratios on the total costs when the demand satisfies the binomial distribution and the variance is 0.5

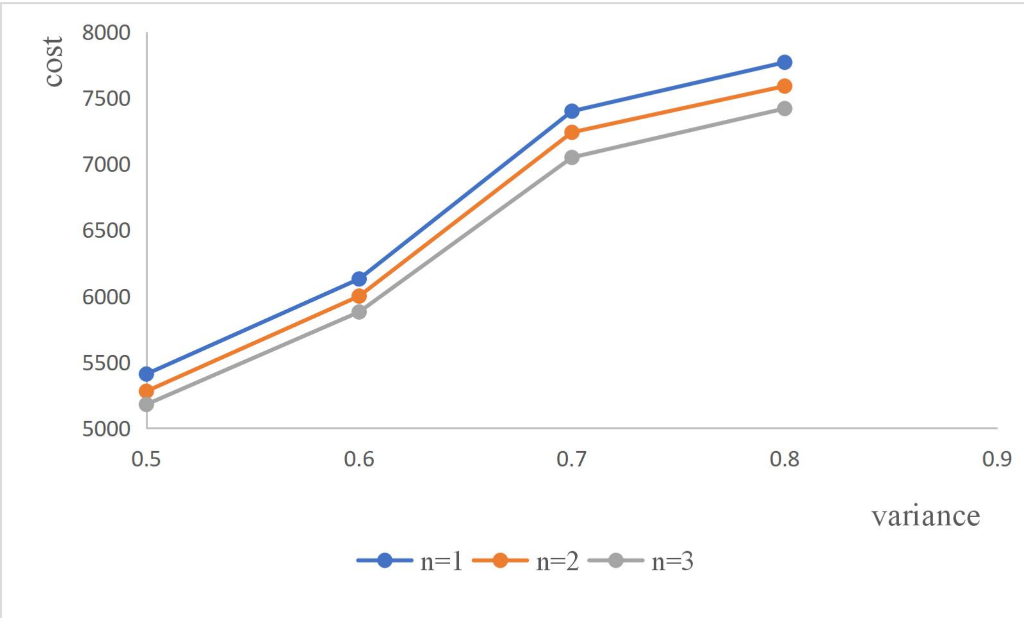

Figure 5: The influence of different matching ratios on the total costs when the demand satisfies the binomial distribution and the mean value is 30

We can draw the following conclusions according to the above figures:

1. When the demand satisfies the binomial distribution and the variance remains unchanged value of 0.5, the total cost will increase with the increase of the mean value, but the impact on the total costs is more obvious when the matching ratio is 1, and the difference is tiny when the matching ratio increases from 2 to 3.

2. When the demand satisfies the binomial distribution and the mean value remains unchanged with a value of 30, the total cost will increase when the variances increase, and

with the increase of variance, the influence of different matching ratios on the total costs becomes more and more prominently.

From the above conclusions, we can find that although the different matching ratios have a certain degree of impact on the total costs. However, when the demand situations are different, the impacts on the total costs are also different. In the case of unimodal, bimodal and trimodal, the fluctuation has a greater impact on the unimodal demand, and the bimodal demand was affected more obviously compared with the trimodal. However, when we change the matching ratio, the unimodal is the least affected by the matching ratio, while the bimodal and trimodal are greatly affected by the matching ratio, and when the fluctuation is different, the decrease trend of the bimodal and trimodal also differs. when the fluctuation is slow, the decreasing trend is nonlinear, while when the fluctuation is sharp, the decreasing trend is almost linear. In addition, when the demand satisfies the Poisson distribution, changing the matching ratio has little effect on the total costs under the same mean value. When the demand satisfies the binomial distribution with a fixed variance, the total costs are less affected by different matching ratio when changing the mean value. Only when the matching ratio is one, the total costs will differ from the other two situations prominently. On the other hand, when the demand satisfies the binomial distribution with a fixed mean value, changing the variance and the total costs are greatly affected by different matching ratios. What's more, with the increase of variance, the influences of different matching ratios on total costs are more and more significant. This shows that under the condition of binomial distribution, changing variance has a more obvious impact on the influence of different matching ratios on total costs than changing mean value. These conclusions are quite different from those summaries obtained under general conditions. We can determine which matching ratio has a greater impact on the total costs according to the distribution and fluctuation of an actual demand, which also has rich practical guiding significance for us to solve the real problems.

5 Conclusion

This paper mainly describes the current situation of recruitment difficulties in nursing homes, and illustrates the characteristics of nursing work and points out that the unreasonable staffing problem aggravates the recruitment difficulties in nursing homes. Therefore, we puts forward some useful methods to solve the staffing problem in nursing institutions. According to the characteristics of multiple shifts, high time-varying demand, hierarchical and collaborative, a two-stage modeling method was adopted. Genetic algorithm and heuristic rules were respectively used to resolve the problem of shift design in the first stage and hierarchical collaboration in the second stage. Finally, according to different demand fluctuations and distributions, this paper analyzes the impact of different matching ratios on the total costs and carries out a series of tests. The results show that there exists differences between the final experimental conclusions and the general conclusions, which have certain enlightenment and rich significance to solve practical problems. But in the future, we will continue to improve the model and find better methods to further study the problem.

6 Acknowledgments

This work is supported by National Natural Science Foundation of China (71701052), Guangdong Natural Science Foundation (2017C030110189), Shenzhen High-Caliber Personnel Research Start-up Project(2020111), Ordinary University Engineering Technology Development Center Project of Guangdong Province(2019GCZX006), Shenzhen Technology University Teaching Reform Project, Guangzhou Yuexiu science and technology plan project(2017-GX-005).

References

- Alex Bonutti, Sara Ceschia, Fabio De, et al.(2017): Modeling and solving a real-life multi-skill shift design problem. Annals of Operations Research, 252, 365-382.
- Aykin T.(2000): A comparative evaluation of modeling approaches to the labor shift scheduling problem. European Journal of Operational Research, 125(2), 381-397.
- Banu Sungur, Cemal Özgüven, Yasemin Kariper.(2017): Shift scheduling with break windows, ideal break periods, and ideal waiting times. Flexible Services & Manufacturing Journal.
- Dantzig, George B.(1954): A COMMENT ON EDIE'S "TRAFFIC DELAYS AT TOLL BOOTHS". Journal of the Operations Research Society of America, 2(3), 339-341.
- Hernández-Leandro Noberto A, Boyer Vincent, Salazar-Aguilar M.(2019): Angélica. A matheuristic based on Lagrangian relaxation for the multi-activity shift scheduling problem. European Journal of Operational Research, 859-867.
- Lequy Q , Desaulniers G, Solomon M M.(2012): A two-stage heuristic for multi-activity and task assignment to work shifts. Computers & Industrial Engineering, 63(4), 831-841.
- Mehran Hojati.(2018): A greedy heuristic for shift minimization personnel task scheduling problem. Computers and Operations Research, 66-76.
- Musliu N , Schaerf A , Slany W. (2004): Local search for shift design. European Journal of Operational Research, 153(1), 51-64.
- Oezgueven C, Sungur B.(2013): Integer programming models for hierarchical workforce scheduling problems including excess off-days and idle labour times. Applied Mathematical Modelling, 9117-9131.
- P. Pakpoom and P. Charnsethikul,(2018): A Column Generation Approach for Personnel Scheduling with Discrete Uncertain Requirements. 2nd International Conference on Informatics and Computational Sciences (ICICoS), Semarang, Indonesia, 1-6.
- Prot,D, Lapègue,T, Bellenguez-Morineau,O.(2015): A two-phase method for the shift design and personnel task scheduling problem with equity objective. International Journal of Production Research, 53(24), 1-13.
- Sana Dahmen, Monia Rekik, François Soumis.(2020): A two-stage solution approach for personalized multi-department multi-day shift scheduling. European Journal of Operational Research, 1051-1063.
- Seckiner S U, Hadi G, Kurt M.(2007): An integer programming model for hierarchical workforce scheduling problem. European Journal of Operational Research, 183(2), 694-699.
- Turgut Aykin.(1996): Optimal Shift Scheduling with Multiple Break Windows. Management Science, vol.42, no4, 591-602.
- Volland J , Fügener, Andreas, Brunner J O.(2017): A column generation approach for the integrated shift and task scheduling problem of logistics assistants in hospitals. European Journal of Operational Research, 260(1), 316-334.
- Y. Chen, X. Zhang, B. Bian and H. Li.(2019): Optimal Staffing Policy in Commercial Banks Under Seasonal Demand Variation. IEEE Access, vol.7, 121111-121126.