A Two-Stage Production Planning Model for Perishable Products Under Uncertainty

Kin Keung Lai[1], **Ming Wang**[1*]

[1] College of Economics, Shenzhen University, Shenzhen, China

Abstract: - This study addresses the production planning problem for perishable products, in which the cost and shortage of products are minimized subject to a set of constraints such as warehouse space, labor working time and machine time. Using the concept of postponement, the production process for perishable products is differentiated into two phases to better utilize the resources. A two-stage stochastic programming with recourse model is developed to determine the production loading plan with uncertain demand and parameters. A set of data from a toy company shows the benefits of the postponement strategy: these include lower total cost and higher utilization of resources. Comparative analysis of solutions with and without postponement strategies is performed.

Key-Words: - Production planning, Stochastic Programming, Modeling, Perishable

1 Introduction

Items like dairy products, medical products and chemical products cannot be stored for a long time because they rot or can no longer be used. For other items such as computers and mobile phones, sale volumes drop dramatically when a new generation is introduced. Seasonal products like high fashion apparel, Christmas gifts and calendars are sold only below full price after a day or a season. These products are regarded as perishable products. Controlling the inventory of perishable products is crucial. On one hand, the demand for perishable products is time-sensitive. This means that the demand dramatically increases as the day approaches the end of life-cycle, such as Christmas Day. On the other hand, a shortage of perishable products while the products are saleable may result in significant loss of revenue because the perishable products cannot be profitable after a certain day. For instance, in manufacturing industries, people want to buy Christmas gifts in or before December only. However, there is little research that addresses aggregate production planning for perishable products. In order to deal with the production planning under limited resources while facing a dramatic growth in demand, in this study we employ a postponement strategy in production planning for perishable products. Postponement in production planning refers to common intermediate products being manufactured in a first phase, and, according to the differentiating options such as colors, sizes and types, production line activities such as dyeing, compounding, final assembling, packaging and so on are postponed to a second phase until customer orders received [1], [2], [7]. Hence, with a postponement strategy, we determine (1) how many finished products should be produced from raw materials directly (direct production), (2) how many semi-finished products should be produced from raw materials (master production), and (3) how many finished products

[*]**Corresponding author:**
Email address : wmtroy@outlook.com

should be produced from semi-finished products (final assembly) so that the resources can be better utilized to meet the dramatic growth in demand. A well-known real-life postponement example is the redesign of the European DeskJet Printer line by Hewlett Packard, as illustrated by Lee and Billington [5].

However, no research is found to solve aggregate production planning of perishable products under an uncertain environment. The purpose of this study is to develop a stochastic programming model to optimize the production planning problem for perishable products; from this the optimal production plan and workforce level for a medium-term planning horizon is determined with the minimal total costs consisting of the production cost, setup cost, labor cost, inventory cost, hiring cost and lay-off cost, and penalty cost associated with under-fulfillment of realized demand under different economic growth scenarios.

2 Problem Formulation

One of the widely-used formulations for decision making under uncertainty is stochastic programming with recourse. The basic idea of this modeling approach is to formulate the problem in a two-stage setting. In the first stage, a decision is made based on the deterministic parameters. When the uncertainty is realized, a corrective recourse action is then made at the second stage. The objective of two-stage stochastic program is to minimize the total costs associated with the first stage decision and the expected future recourse costs at the second stage. The incorporation of the expected future recourse costs provides a proactive approach to tackle the future uncertainty at the beginning of modeling. For detail discussion of the approach of two-stage stochastic programming with recourse, the reader is referred to Dantzig [3], Kall and Wallace [4] and Ruszczyński and Shapiro [6].

In this study, the aggregate production planning problem for perishable products faced by a toy company in Hong Kong is investigated. For cost effectiveness, the decision makers have to determine the quantity of product i, i = 1,2,…,n, manufactured over each period of time t, t = 1,2,…T, to fulfill market demands under different scenarios s, s = 1,2,…,S. The production loading plan consists of: (1) the quantity of finished products to be produced from raw materials directly (direct production), (2) the quantity of semi-finished products to be produced from raw materials (master production), and (3) the quantity of finished products to be produced from semi-finished products (final assembly) in each period of time.

2.1 Notation

Parameters:

First-stage parameters:

C_{KXi}: the setup cost for producing finished product i from raw materials

C_{KYi}: the setup cost for producing semi-finished product i from raw materials

C_{KZi}: the setup cost for producing finished product i from semi-finished products

C_{Wt}: the labor cost in period t

C_{Ht}: the cost to hire one worker in period t

C_{Lt}: the cost to lay-off one worker in period t

$\overline{W_t}$: the maximum number of workers available in period t

Second-stage parameters:

D_{it}^{s}: the forecast demand for product i in period t under scenario s

C_{PXi}^{s}: the regular-time unit production cost to produce one unit of finished product i from raw materials under scenario s
C_{PYi}^{s}: the regular-time unit production cost to produce one unit of semi-finished product i from raw materials under scenario s
C_{PZi}^{s}: the regular-time unit production cost to produce one unit of finished product i from semi-finished products under scenario s
C_{OXi}^{s}: the overtime unit production cost to produce one unit of finished product i from raw materials under scenario s
C_{OYi}^{s}: the overtime unit production cost to produce one unit of semi-finished product i from raw materials under scenario s
C_{OZi}^{s}: the overtime unit production cost to produce one unit of finished product i from semi-finished products under scenario s
$C_{\alpha i}^{s}$: the inventory holding cost for one unit of finished product i under scenario s
$C_{\beta i}^{s}$: the inventory holding cost for one unit of semi-finished product i under scenario s
C_{Ui}^{s}: the cost of under-fulfillment for one unit of finished product i under scenario s
a_{Xi}: the man hours required to produce one unit of finished product i from raw materials
a_{Yi}: the man hours required to produce one unit of semi-finished product i from raw materials
a_{Zi}: the man hours required to produce one unit of finished product i from semi-finished products
b_{Xi}: the machining time required to produce one unit of finished product i from raw materials
b_{Yi}: the machining time required to produce one unit of semi-finished product i from raw materials
b_{Zi}: the machining time required to produce one unit of finished product i from semi-finished products
δ: the regular working hours of labor in each period
λ_{t}^{W}: the fraction of regular workforce available

for over-time in period t
λ_{t}^{M}: the fraction of regular machine capacity available for over-time use in period t
M_{t}: the maximum regular time machine capacity in period t
$v_{\alpha i}$: the space occupied by one unit of finished product i
$v_{\beta i}$: the space occupied by one unit of semi-finished product i
$\overline{I_{t}}$: the storage space limitation in period t

Decision variables:
First-stage decision variables:
K_{Xit}: the indicator for producing finished product i from raw materials in period t (if $K_{Xit} = 1$, then $P_{Xit} > 0$; if $K_{Xit} = 0$, then $P_{Xit} = 0$)
K_{Yit}: the indicator for producing finished product i from raw materials in period t (if $K_{Yit} = 1$, then $P_{Yit} > 0$; if $K_{Yit} = 0$, then $P_{Yit} = 0$)
K_{Zit}: the indicator for producing finished product i from raw materials in period t (if $K_{Zit} = 1$, then $P_{Zit} > 0$; if $K_{Zit} = 0$, then $P_{Zit} = 0$)
H_{t}: the number of workers hired in period t
L_{t}: the number of workers laid-off in period t
W_{t}: the number of workers in period t

Second-stage decision variables:

P_{Xit}: the number of finished products i produced from raw materials during regular time in period t

P_{Yit}: the number of semi-finished products i produced from raw materials during regular time in period t

P_{Zit}: the number of finished products i produced from semi-finished products during regular time in period t

O_{Xit}: the number of finished products i produced from raw materials during overtime in period t

O_{Yit}: the number of semi-finished products i produced from raw materials during overtime in period t

O_{Zit}: the number of finished products i produced from semi-finished products during overtime in period t

$I_{\alpha it}^{s}$: the inventory level of finished product i in period t under scenario s

$I_{\beta it}^{s}$: the inventory level of semi-finished product i in period t under scenario s

U_{it}^{s}: the under-fulfillment of finished product i in period t under scenario s

2.2 Objective function

The objective function at the first stage:

$$Min \sum_{t=1}^{T}\sum_{i=1}^{n}(C_{KXi}K_{Xit} + C_{KYi}K_{Yit} + C_{KZi}K_{Zit}) + \sum_{t=1}^{T} C_{Wt}W_t + \sum_{t=1}^{T}(C_{Ht}H_t + C_{Lt}HL_t) \quad (1)$$

The first term in expression (1) is the setup cost. The second term is the labor cost, which is associated with regular-time workers. The last term is total hiring and laying-off cost associated with changes in the workforce level.

The objective function at the second stage:

$$Min \sum_{s=1}^{S} P_s \left[\sum_{t=1}^{T}\sum_{i=1}^{n}(C_{PXi}^{s}P_{Xit} + C_{PYi}^{s}P_{Yit} + C_{PZi}^{s}P_{Zit}) + \sum_{t=1}^{T}\sum_{i=1}^{n}(C_{OXi}^{s}O_{Xit} + C_{OYi}^{s}O_{Yit} + C_{OZi}^{s}O_{Zit}) + \sum_{t=1}^{T}\sum_{i=1}^{n}(C_{\alpha i}^{s}I_{\alpha it}^{s} + C_{\beta i}^{s}I_{\beta it}^{s} + C_{Ui}^{s}U_{it}^{s})\right] \quad (2)$$

The first term in expression (2) is the regular-time production cost, which comprises associated direct production, master production and final assembly. The second term is the over-time production cost, which is associated with direct production, master production and final assembly. The third term is the inventory cost associated with the storage of units of finished products and semi-finished products in warehouses for a period of time. The last term is the penalty cost associated with under-fulfillment of demand.

2.3 Constraints

The constraints at the first stage:

$$W_t = W_{t-1} + H_t - L_t, t = 1,2,\dots,T \quad (3)$$

$$W_t \leq \overline{W_t}, t = 1,2,\dots,T \quad (4)$$

$$W_t, H_t, L_t \geq 0, i = 1,2,\dots,n, t = 1,2,\dots,T \quad (5)$$

$$K_{Xit}, K_{Xit}, K_{Xit} = \{0,1\}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (6)$$

Constraint (3) ensures that the available workforce in any period equals the workforce from the previous period plus any change in workforce level during the current period. The change in workforce level may be due to either hiring extra workers or laying-off redundant workers. It is noted that $H_t * L_t = 0$ because either the net hiring or the net laying-off of workers takes place over a period, but not both. Constraint (4) ensures the upper-bounds of change in workforce level over a period are provided. Constraint (5) ensures that all decision variables are non-negative. Boolean constraints (6) are used for the setup indications of the production activities.

The constraints at the second stage:

$$I_{\alpha it}^{s} - U_{it}^{s} = I_{\alpha it-1}^{s} + P_{Xit} + O_{Xit} + P_{Zit} + O_{Zit} - D_{it}^{s}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (7)$$

$$I_{\beta it}^{s} = I_{\beta it-1}^{s} + P_{Yit} + O_{Yit} - P_{Zit} - O_{Zit}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (8)$$

$$\sum_{i=1}^{n} (v_{\alpha i} I_{\alpha it}^{s} + v_{\beta i} I_{\beta it}^{s}) \le \overline{I_t}, t = 1,2, \dots, T \quad (9)$$

$$\sum_{i=1}^{n} (a_{Xi} P_{Xit} + a_{Yi} P_{Yit} + a_{Zi} P_{Zit}) \le \delta W_t, t = 1,2, \dots, T \quad (10)$$

$$\sum_{i=1}^{n} (a_{Xi} O_{Xit} + a_{Yi} O_{Yit} + a_{Zi} O_{Zit}) \le \lambda_t^M \delta W_t, t = 1,2, \dots, T \quad (11)$$

$$\sum_{i=1}^{n} (b_{Xi} P_{Xit} + b_{Yi} P_{Yit} + b_{Zi} P_{Zit}) \le M_t, t = 1,2, \dots, T \quad (12)$$

$$\sum_{i=1}^{n} (b_{Xi} O_{Xit} + b_{Yi} O_{Yit} + b_{Zi} O_{Zit}) \le \lambda_t^M M_t, t = 1,2, \dots, T \quad (13)$$

$$P_{Xit} + O_{Xit} \le \Pi K_{Xit}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (14)$$

$$P_{Yit} + O_{Yit} \le \Pi K_{Yit}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (15)$$

$$P_{Zit} + O_{Zit} \le \Pi K_{Zit}, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (16)$$

$$I_{\alpha it}^{s}, I_{\beta it}^{s}, U_{it}^{s}, P_{Xit}, P_{Yit}, P_{Zit}, O_{Xit}, O_{Yit}, O_{Zit} \ge 0, i = 1,2, \dots, n, t = 1,2, \dots, T \quad (17)$$

where Π is a large positive number.

Constraint (7) determines either the quantity of finished products stored in the warehouse or the shortfall in meeting market demand. Constraint (8) determines the quantity of semi-finished products stored in the warehouse. The total quantity of semi-finished products produced at the company's plants during period t plus previous stock at period t–1 must equal the semi-finished products stored in the warehouse at period t plus the quantity of semi-finished products used to perform final assembly. The physical storage space at period t is limited by constraint (9). Constrains (10) and (11) limit the labor working hours during regular time and overtime respectively. Similarly, Constrains (12) and (13) limit the machining time during regular time and overtime respectively. Constraints (14) – (16) ensure that setup costs will be incurred when the corresponding production activities started. Constraint (17) ensures that the second-stage decision variables are non-negative.

3 Problem Solution

In order to illustrate the flexibility of the proposed stochastic programming approach for aggregate production planning problem for Christmas products, we use the data provided by the plush toy company in Hong Kong. The tactical/operational level of decision-making in the production planning process is described below. Based on the company's projection report, a two-month planning horizon is determined (November and December).

The company receives sales orders from its sales branches covering America and Europe. Each order may require two type of products, i = 1,2 covering 8 weeks, t = 1,2,…8. It is assumed that future economic scenarios will fit into one of four possible scenarios – boom, good, fair and poor – with associated probabilities of 0.40, 0.25, 0.20 and 0.15 respectively.

The setup costs as well as labor and machine requirements of different production activities are given in Table 1. Table 2 shows the limitations on workforce level, machine capacity, overtime production and warehouse spaces in each period. Table 3 lists regular time labor cost, and hiring and laying-off costs associated with changes in the workforce level. The unit space occupation for finished products and semi-finished products are provided in Table 4. The production cost, inventory cost and shortage cost are shown in Tables 5–7. It is noted that, owing to the characteristics of the products, the shortage cost is time-sensitive and dramatically increases as the time approaches the event kick-off period (i.e. the ending period). For each weekly period, the product quantities required under different economic scenarios for the market are shown in Table 8.

The production loading plan with postponement strategy is shown in Table 9. It can be seen that the majority of products are produced using regular-time labor. In order to meet the growth of demand in the last two periods, production management is recommended to produce semi-finished products in periods 4–6 and perform final assembly in periods 7 and 8. The majority of resources consumed in period 8 are used to perform final assembly of product 2. It is shown that, using the postponement strategy, more products can be produced, particularly in period 8. Lastly, the workforce level in each period attains the upper-bound limit. The corresponding number of workers hired and laid off can also be found in Table 9.

The production loading plan without postponement strategy is also shown in Table 9. Compared with the production loading plan with postponement strategy, it is noted that the company produces more finished products and stores them in periods 5 and 6 for the demand in December. Since the storage of finished products incurs higher inventory cost and takes up more warehouse space, the production planning without postponement strategy is not a preferable for production management. Therefore, one of the advantages of postponement strategy is that, without adding extra costs and resources such as machine capacity, workforce level and warehouse space, more products can be produced in December with postponement strategy. This strategy is more attractive for production management.

The breakdown of costs incurred for production plans with and without postponement strategy is listed in Table 10. For the production planning with postponement strategy, the operational cost, which is the sum of production cost, setup cost, labor cost, inventory cost, and hiring cost and lay-off cost, is $10,400,305. Clearly, when the demand requirements are smaller than the available production (from previous inventory and current production) the stock will be kept at the end of each particular period t under scenario s, and the corresponding inventory cost will be incurred. On the other hand, when the demand requirements are not satisfied, the company's service level and goodwill will be damaged. Compensation may be considered to cover the excess demand. This compensation is considered as a penalty cost. Table 10 shows that, under the optimal production loading plan, the penalty cost is $4,758,366. Overall, the total cost, which is the sum of operational cost and penalty cost, is $15,158,671.

Originally, under the present strategy (without postponement) the total cost incurred is $16,248,222. In comparison with the present strategy (without postponement) a saving of about 6.7% is made by following the proposed strategy (with postponement).

4 Conclusion

In this study, a stochastic programming approach for the aggregate production planning problem for perishable products with uncertain demand is proposed. The computation results obtained from a set of real-world data show that the proposed model is practical for dealing with uncertain economic scenarios. It is believed that the model can provide a credible and effective methodology for real-world production planning problems in an uncertain environment. However, there is still room for improvement and investigation. First, real data from companies can be used to validate the model and to analyze its sensitivity to changes in production planning strategies. Second, sensitivity analysis may be conducted on the cost parameters in the objective function to test the trade-off between total cost and shortage costs. Third, the selection of probability distribution of economic scenarios could be further investigated. Finally, the whole area of study associated with segregating market demand by region/country, and including different selling prices by region/country, can offer scope for making the APP a more useful basis for decision-making, in which we are not simply minimizing costs of production, etc., but are maximizing profit.

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Table 1. Operating and costs data (in HK$, 1US$ = 7.8HK$).

	Product	Direct finished product production	Semi-finished product production	Transfer production
Setup cost ($)	1	2000	1000	1500
	2	2500	1000	2000
Labor time (hour)	1	0.5	0.35	0.15
	2	0.6	0.35	0.25
Machining time (hour)	1	0.5	0.4	0.1
	2	0.6	0.4	0.2

Table 2. Warehouse, machine and workforce capacity.

Warehouse space limitation, $\overline{I_{\iota t}}$	1,000 m^3
Maximum workforce level, $\overline{W_t}$	1,000
Maximum machine capacity, M_t	16,000
Fraction of workforce available for over-time, λ_t^W,	0.3
Fraction of machine capacity available for over-time, λ_t^M,	0.4

Table 3. Labor costs and hiring and laying-off costs (in HK$).

	Period							
	1	2	3	4	5	6	7	8
Labor cost per worker per period, C_t^W($)	80	80	80	80	80	80	80	80
Hiring cost per worker, C_t^H($)	80	80	100	100	100	80	80	80
Laying-off cost per worker, C_t^L($)	120	120	120	120	120	120	120	120

Table 4. Warehouse space occupation.

Product	Finished product warehouse space occupied (m^3)	Semi-finished product warehouse space occupied (m^3)
1	1.0	0.3
2	1.0	0.3

Table 5. Production costs under different scenarios (in HK$).

Production cost		Product, i	Scenario, s Boom	Good	Fair	Poor
Direct finished product production	Regular time	1	60	55	53	50
		2	70	65	63	60
	Overtime	1	60	55	53	50
		2	70	65	63	60
Semi-finished product production	Regular time	1	40	35	33	30
		2	40	35	33	30
	Overtime	1	40	35	33	30
		2	40	35	33	30
Transfer production	Regular time	1	40	35	33	30
		2	50	45	43	40
	Overtime	1	40	35	33	30
		2	50	45	43	40

Table 6. Inventory costs under different scenarios (in HK$).

Inventory cost	Product, i	Scenario, s Boom	Good	Fair	Poor
Direct finished product production	1	60	55	53	50
	2	60	55	53	50
Semi-finished product production	1	15	10	8	5
	2	15	10	8	5

Table 7. Shortage costs under different scenarios (in HK$).

		Period, t							
Product, i	Scenario, s	1	2	3	4	5	6	7	8
1	Boom	400	440	484	532	584	644	708	780
	Good	300	330	363	399	438	483	531	585
	Fair	260	286	315	346	380	419	460	507
	Poor	200	220	242	266	292	322	354	390
2	Boom	480	528	580	640	716	772	852	936
	Good	360	396	435	480	537	579	639	702
	Fair	312	343	377	416	465	502	554	608
	Poor	240	264	290	320	358	386	426	468

Table 8. Market demand data.

		Period, t							
Product, i	Scenario, s	1	2	3	4	5	6	7	8
1	Boom	4000	4400	5000	5800	6800	8800	12600	23800
	Good	3000	3300	3750	4350	5100	6600	9450	17850
	Fair	2600	2860	3250	3770	4420	5720	8190	15470
	Poor	2000	2200	2500	2900	3400	4400	6300	11900
2	Boom	6400	6800	7600	8400	9600	11400	14200	20200
	Good	4800	5100	5700	6300	7200	8550	10650	15150
	Fair	4160	4420	4940	5460	6240	7140	9230	13130
	Poor	3200	3400	3800	4200	4800	5700	7100	10100

Table 9. Production loading plans with and without postponement strategy.

With postponement strategy

		Product, *i*	Period, *t* 1	2	3	4	5	6	7	8
Direct finished product production	Regular time	1	2000	2860	1132	3770	4420	6600	7407	6443
		2	3200	2655	4940	5460	2240	7833	6650	0
	Overtime	1	0	0	2118	0	0	0	0	4800
		2	0	1765	0	0	4000	717	4000	0
Semi-finished product production	Regular time	1	0	0	0	0	8650	0	0	0
		2	0	0	0	804	4053	0	0	0
	Overtime	1	0	0	0	0	0	0	0	0
		2	0	0	0	4665	0	5629	0	0
Transfer production	Regular time	1	0	0	0	0	0	0	2043	6607
		2	0	0	0	0	0	0	0	15150
	Overtime	1	0	0	0	0	0	0	0	0
		2	0	0	0	0	0	0	0	0
Workforce level			441	441	441	680	1000	1000	1000	1000
Hiring			0	0	0	239	320	0	0	0
Laying-off			59	0	0	0	0	0	0	0

Without postponement strategy

		Product, *i*	Period, *t* 1	2	3	4	5	6	7	8
Direct finished product production	Regular time	1	2000	2860	3250	3770	0	5720	8190	13050
		2	3200	3500	3175	3475	1333	8567	6508	5428
	Overtime	1	0	0	0	0	4420	0	0	4800
		2	0	920	1765	1985	317	4000	4000	0
Workforce level			441	441	441	496	1000	1000	1000	1000
Hiring			0	0	0	55	504	0	0	0
Laying-off			59	0	0	0	0	0	0	0

Table 10. Breakdown of costs (in HK$).

	Production cost	Setup cost	Labor cost	Inventory cost	Hiring and laying-off	Operational cost	Penalty cost	Total cost
With postponement	6,695,583	42,500	480,323	3,118,974	62,925	10,400,305	4,758,366	15,158,671
Without postponement	6,177,306	36,000	465,600	4,018,201	62,925	10,760,032	5,488,190	16,248,222